# Supplementary materials

## 1. The four sets of subtractions used in the choice and no-choice conditions

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| **Series 1** |  |  | **Series 2** |  |  | **Series 3** |  |  | **Series 4** |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 602 - 313 =  | 289 |  | 502 - 263 = | 239 |  | 504 - 258 = | 246 |  | 914 - 886 =  | 28 |
| 714 - 687 = | 27 |  | 913 - 887 = | 26 |  | 912 - 884 = | 28 |  | 503 - 17 =  | 486 |
| 502 - 18 =  | 484 |  | 806 - 28 = | 778 |  | 603 - 16 = | 587 |  | 714 - 347 =  | 367 |
| 914 - 447 = | 467 |  | 713 - 368 =  | 345 |  | 814 - 396 = | 418 |  | 906 - 18 =  | 888 |
| 606 - 588 = | 18 |  | 902 - 18 = | 884 |  | 704 - 677 = | 27 |  | 613 - 296 = | 317 |
| 913 - 27 =  | 886 |  | 814 - 397 = | 417 |  | 816 - 28 =  | 788 |  | 802 - 776 =  | 26 |
| 504 - 476 = | 28 |  | 703 - 686 = | 17 |  | 603 - 294 =  | 309 |  | 714 - 27 =  | 687 |
| 814 - 26 = | 788 |  | 512 - 24 = | 488 |  | 514 - 26 =  | 488 |  | 802 - 413 = | 389 |
| 704 - 358 =  | 346 |  | 604 - 577 = | 27 |  | 804 - 787 = | 17 |  | 513 - 486 =  | 27 |
| 812 - 788 = | 24 |  | 713 - 27 = | 686 |  | 716 - 368 =  | 348 |  | 904 - 446 =  | 458 |
| 704 - 17 = | 687 |  | 802 - 788 = | 14 |  | 903 - 27 =  | 876 |  | 613 - 587 =  | 26 |
| 512 - 248 = | 264 |  | 916 - 448 =  | 468 |  | 612 - 588 =  | 24 |  | 802 - 24 =  | 778 |
| 903 - 886 = | 17 |  | 603 - 26 = | 577 |  | 702 - 18 =  | 684 |  | 513 - 268 =  | 245 |
| 616 - 28 = | 588 |  | 514 - 486 = | 28 |  | 514 - 488 =  | 26 |  | 604 - 28 =  | 576 |
| 803 - 408 =  | 395 |  | 602 - 293 = | 309 |  | 902 - 463 =  | 439 |  | 703 - 687 =  | 16 |

## 2. Teacher questionnaire (translated from Dutch)

**1. Calculation methods**

* Which calculation method do you use in the classroom?
* Do you supplement this calculation method with other methods or materials? If yes, which ones?

**2. Subtraction problems with numbers > 100, with crossing over tens, hundreds and/or thousands.**

**Example: 526 – 348 = ?**

* Do you teach/remediate the curriculum concerning mentally solving subtraction problems with numbers > 100, with crossing over tens, hundreds and/or thousands?
* Which strategies are used to teach children to solve subtraction problems with numbers > 100, with crossing over tens, hundreds and/or thousands? Give an example by solving 425 – 317 and 1051 – 988.
* Are children allowed to use strategies other than the ones taught? If yes, please provide an example of such a strategy.
* Are mathematically higher and lower achieving children taught the same strategies? If not, indicate the differences.

**3. Point sums**

**Example: 25 + . = 43 of 396 + . = 524**

* Do you teach/remediate point sums in your class?
* In which context are point sums used (word problems, mental calculation…)? Please provide an example for every context.
* Are both mathematically higher and lower achieving children taught how to solve point sums? If not, indicate the differences.

**4. Direct subtraction versus subtraction by addition**

Subtraction exercises can be solved in various ways and with different mental calculation strategies. One way to classify these strategies is based on the main operation being performed. This classification discerns to two types of strategies:

* Direct subtraction: the difference is determined by *taking away* the second term from the first term, e.g., solving…

31 – 17 = ? via 31 – 10 – 7 = 21 – 7 = 14

821 – 475 = ? via 821 – 400 – 75 = 421 – 70 – 5 = 351 – 5 = 346

* Subtraction by addition: the difference is determined by *counting on* from the second to the first term, e.g., solving…

31 – 17 = ? via 17 + 3 = 20; 20 + 11 = 31; thus the answer is 3 + 11 = 14

821 – 475 = ? via 475 + 25 = 500; 500 + 321 = 821; thus the answer is 25 + 321 = 346

* Do you teach/remediate the subtraction by addition strategy in your class?
* How is this strategy taught?
* Do children learn to apply this strategy to all sorts of exercises or only to exercises with specific characteristics (such as “202 - 199 = ?” Where there is an extremely small difference between minuend and subtrahend)?
* Are both mathematically higher and lower achieving children taught how to use the subtraction by addition strategy? If not, indicate the differences.

## 3. Script used during data collection (translated from Dutch)

### **First session**

#### Instructions choice-decide condition

Today we’re going solve subtractions up to 1000 on the computer. I am very interested in the way you solve the subtraction exercises. That is why you have to explain very clearly to me after each exercise which way you would use to solve the exercise. You can choose between two different strategies. I'm going to explain those two strategies to you in a moment and then we'll make some example exercises together.

I am now going to make some example exercises with you and explain to you the two strategies you can use to solve these exercises. Then I'm going to let you solve 15 exercises.

Computer displays a screen saying “welcome” and the assignment for the children, namely to indicate the strategy they would use to solve the exercises.

**1st example exercise: 712 - 346 = ? (solve via DS - explanation by test leader)**

I'm going to explain to you two strategies to solve this exercise.

Computer displays a figure on which the DS strategy is explained.

A first strategy you can use to solve the exercise is by subtracting the hundreds (H), the tens (T) and the units (E) of the smallest number step by step from the largest number. From the largest number, 712 *(examiner points with a pen to the largest number)*, you first subtract the H, *(examiner points with a pen to the H in the smallest number)*, namely 300, so 712 - 300. Then you have 412 left. From those 412 you then subtract the T of the smallest number *(examiner points with a pen to the T in the smallest number)*, namely 40; 412 - 40. This will bring you to 372. Finally, from 372 you subtract the E of the smallest number *(examiner with a pen points to the E in the smallest number)*, namely 6, and you arrive at 366. Do you understand how this strategy works?

[If the student does not understand this strategy, the same exercise is solved again using the DS strategy]

**2nd example exercise: 712 - 346 = ? (solve via SBA - explanation by test leader)**

Computer displays a figure on which the SBA strategy is explained.

A second strategy that you can use, is not to start with the largest, namely 712, but with the smallest number, namely 346 *(examiner points to the smallest number with a pen)*. So you wonder how much you have to add to 346 to get to 712 (examiner points with a pen first tot the smallest and then to the largest number). First you have to calculate how much you have to add to the smallest number to arrive at the next H, this is the round hundred that is greater than 346 and is very close to it, here it is 400. To get from 346 to 400 you have to add 54, because 346 + 54 = 400. You have already made it to the next hundred, but that is not enough. You now have to get from 400 to the largest number, 712. To do this, you have to add 312 to 400, thus arriving at 712. Afterwards you add the numbers you have obtained using this strategy. So you add up the number you used to get from 346 to 400, which was 54, and the number you used to get from 400 to 712, which was 312, together. When you add 54 to 312, you arrive at 366. Do you understand this strategy?

[If the student does not understand this strategy, the same exercise is solved again using the SBA strategy]

In a moment I will give you exercises that you can solve using one of the strategies that I have just explained. However, I am not going to ask you to solve those exercises completely. I'm just going to ask you to tell me what strategy you would use to solve the exercise. So you don't really have to calculate the exercise with one of those two strategies, you just have to tell me which strategy you would use. Whether you would use the first strategy, and thus subtract the smallest number from the largest number step by step, or use the second strategy, and thus start with the smallest number and then consider how much you have to add to the largest number.

A name for each strategy is agreed upon with the child. If no proposal emerges from the child, the strategies were named “adding” (SBA) and “taking away” (DS).

It is very important that for the exercises that I am going to give you, you will either use the first strategy, i.e., the taking away strategy, and thus subtract the smallest number step by step from the largest number, or use the second strategy, i.e., the adding strategy, thus start with the smallest number and determine how much you have to add to get to the largest number. Please note: you do not have to calculate or give a final answer, but you do have to tell which strategy you would use to solve this exercise.

Computer displays a screen showing the different steps used to present each item and when the child should answer, namely fixation cross 🡪 presentation of exercise 🡪 indicating strategy.

In a moment you will first see a fixation cross on the screen *(test leader points to the fixation cross on the screen with a pen)*. Right behind that, the exercise appears at the same place. For this item you verbally tell me which strategy you would use to solve it. You should not solve this exercise, but you have to tell me which method you would use to solve it.

We will start with an example exercise so that I can see whether you have understood the method, afterwards we start with the real exercises.

**3rd example exercise: 521 - 245 = ? (child indicates preferential strategy - DS or SBA)**

A fixation cross appears on the computer, then the exercise is displayed on the computer screen. When the student names his / her preferential strategy (adding or taking away), the researcher presses the spacebar to measure the reaction time. This also removes the exercises from the screen. Subsequently, the researcher registers the student's strategy on the computer, and whether the reaction time was measured correctly.

**Last instruction before the start of the actual test**

Computer displays the figures on which the SBA and DS strategy are explained.

Have you understood everything correctly before we start the real exercises? It is very important that you do not solve the exercise, but that you just tell me the strategy you would use to solve the exercise.

Do you have any questions?

A screen appears indicating that the real test will begin. The child completes a set of exercises in which he / she only indicates the strategy he / she would use to solve each exercise. The researcher stops the time for each item when the student indicates his / her preferential strategy.

#### Instructions choice-compute condition

During the next series of exercises you will have to solve the exercises one by one as quickly and as correctly as possible. I am very interested in the way you solve these exercises. That is why you have to explain to me very clearly after each exercise how you solved the exercise. You can choose between the two different strategies I already explained. I will repeat these strategies with you, using some example exercises. You will not be allowed to use a pen or paper when solving the exercises. After each exercise you will have to explain to me how you arrived at your answer. I am now going to make some example exercises with you and explain to you the two strategies you can use to solve each exercise. Then I'm going to let you solve 15 exercises.

Computer displays a screen saying “welcome” and the assignment for the children, namely to solve the exercises as correctly and quickly as possible.

**1st example exercise: 712 - 346 = ? (solve via DS and SBA - explanation by test leader)**

*[The test leader uses the same examples as used in the instructions for the choice-decide condition to explain both strategies to the child, using the exact same procedure. When children indicate they do not understand one of the strategies, the same exercise is solved again using the strategy that was unclear for the child.]*

It is very important that in the exercises that I am going to give you, you will either use the first strategy, i.e., the taking away strategy, and thus subtract the smallest number step by step from the largest number, or use the second strategy, i.e., the adding strategy, thus start with the smallest number and determine how much you have to add to get to the largest number.

Computer displays a screen showing the different steps used to present each item and when/how the child should answer, namely fixation cross 🡪 presentation of exercise 🡪 indicating strategy.

In a moment you will first see a fixation cross on the screen *(test leader points to the fixation cross on the screen with a pen)*. Right behind that, the exercise appears at the same place. You have to solve each exercise in your head as quickly and correctly as possible. If you know the answer, you can say it out loud. I will then register your answer on the PC. Afterwards a screen will appear with the question: How did you solve this exercise? You will then explain to me how you solved the exercise, you do this by telling me which intermediate steps you have used.

I'm now going to ask you to solve a sample exercise with both the first and the second strategy. In this way I can check whether you have understood the two strategies before we start solving the real exercises.

During the choice-decide condition, a name for each strategy has been agreed upon with the child to identify these two strategies. These names are reused.

**2nd example exercise: 521 - 245 = ? (solved by child using DS)**

Computer displays a figure on which the DS strategy is explained.

First, I ask you to solve an exercise using the first strategy. So you start with the largest number and you subtract the smallest. First you subtract the H, then the T, then the E. You will first solve the exercise in your head and when you know the answer, you tell me the solution out loud. Afterwards you will tell me which steps you have used. That way I can check if you have understood this strategy correctly. Are you ready?

A fixation cross appears on the screen, followed by the exercise. When the student answers, the researcher presses space to measure the reaction time. This also removes the exercise from the screen. The examiner then registers the student's answer on the computer, and whether the reaction time was measured correctly. Finally, the problem reappears on the screen, along with "How did you solve this exercise?". The subject then verbally explains which intermediate steps he / she has used. The examiner registers the appropriate code for the used strategy (DS in this example). If the solution method is worded correctly by the child, the researcher indicates the child executed the strategy correctly. If necessary, the strategy execution is corrected and explained again to the child.

**3rd example exercise: 521 - 245 = ? (solved by child using SBA)**

*[The same procedure as described in the previous example is used, now using the SBA strategy.]*

**Last instruction before the start of the actual test**

Computer displays the figures on which the SBA and DS strategy are explained.

Have you understood everything correctly before we start the actual test? I repeat the 2 most important things for you.

* First, it is very important that you try to solve each exercise as quickly and correctly as possible. Definitely don't try to guess the solution, but use one of the two strategies I explained to solve the exercises.
* Second, after you have solved an exercise, I'm going to ask you each time how you solved the exercise. You will then explain which intermediate steps you have used to arrive at your solution.

Do you have any questions?

A screen appears indicating that the real test will begin. The child solves a set of exercises in which he / she chooses the strategy he / she would use to solve each exercise. After solving each exercise, the child explains the intermediate steps he / she took to solve the exercise. The researcher deduces the strategy the child used based on the given description.

### **Second session**

*[The order of both no-choice conditions was randomized for all children]*

#### Instructions DS no-choice condition

*[Depending on the order of both no-choice conditions, the researchers starts by saying…]*

* You solved some exercises last time where you were allowed to choose between two strategies to solve each exercise.
* You just solved some exercises using the second strategy, i.e., the adding strategy.

Computer displays a screen saying “welcome” and the assignment for the children, namely to solve the exercises as correctly and quickly as possible.

**1st example exercise: 712 - 346 = ? (solve via DS - explanation by test leader)**

This time you are going to solve all the exercises using the first strategy, i.e., taking away that I explained in the previous session. We will practice this strategy again before we start the actual test.

Computer displays a figure on which the DS strategy is explained.

*[The researcher explains how to use the DS strategy, identical to how it was done in the first session. When children indicate they do not understand the DS strategy, the same exercise is solved again.]*

Computer displays a screen showing the different steps used to present each item and when/how the child should answer, namely fixation cross 🡪 presentation of exercise 🡪 indicating strategy.

In a moment you will first see a fixation cross on the screen *(test leader points to the fixation cross on the screen with a pen)*. Thereafter, the exercise appears on the screen. You have to solve each exercise in your head as quickly and correctly as possible. If you know the answer, you can say it out loud. I will then register your answer on the PC. Afterwards a screen will appear with the question: How did you solve this exercise? You will then explain to me how you solved the exercise, you do this by telling me which intermediate steps you have used to become your solution.

I'm going to let you solve one more example exercise to remind you how the procedure goes.

**2nd example exercise: 634 - 378 = ? (solved by child using DS)**

Computer displays a figure on which the DS strategy is explained.

Are you ready?

A fixation cross appears on the screen, followed by the exercise. When the student answers, the researcher presses space to measure the reaction time. This also removes the exercise from the screen. The examiner then registers the student's answer on the computer, and whether the reaction time was measured correctly. Finally, the problem reappears on the screen, along with "How did you solve this exercise?". The subject then verbally explains which intermediate steps he / she has used. The examiner registers the appropriate code for the used strategy (DS in this example). If the DS strategy is worded correctly by the child, the researcher indicates the child executed the strategy correctly. If necessary, the strategy execution is corrected and explained again to the child.

**Last instruction before the start of the actual test**

Computer displays the figures on which the DS strategy is explained.

Have you understood everything correctly before we start the actual test? I repeat the 3 most important things for you.

* First, it is very important that you try to solve each exercise as quickly and correctly as possible. Definitely don't try to guess the solution.
* Second, it is very important that you solve the exercises using the taking away strategy. So you start with the largest number and then you subtract the smallest number from it step by step.
* Third, after you have solved an exercise, I'm going to ask you each time how you solved the exercise. You will then explain which intermediate steps you have used to arrive at your solution.

Do you have any questions?

A screen appears indicating that the real test will begin. The child solves a set of exercises using the DS strategy. After solving each exercise, the child explains the intermediate steps he / she took to solve the exercise. The researcher deduces whether the child used the correct strategy based on the given description.

#### Instructions DS no-choice condition

*[The same procedure as described in the previous no-choice condition is used, now using the SBA strategy. First the researcher explains the SBA strategy by solving 712 – 346, then the child is asked to solve 634 – 378 using SBA. Finally, the actual test is conducted]*