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Impact Sheet

Van Der Auwera, S., Mathys, L., De Smedt, B., Torbeyns, J. & Verschaffel, L. (2021). Upper Elementary School Children's Adaptive Use of Subtraction by Addition: A Choice/No-Choice Replication Study Involving Two Choice Conditions. *Implementation and Replication Studies in Mathematics Education*, *1*(1), 111–138 DOI: 10.1163/26670127-01010005.

1 Problem Addressed

Over the last years there has been a turn from a mathematical instruction mainly aimed at learning to solve mathematical exercises through the standard strategies taught at school (i.e., routine expertise) towards a more conceptually-based, process-based, and problem-based mathematical instruction. The focus has shifted towards adaptive expertise, i.e. the use of various strategies to solve mathematical exercises efficiently and adaptively.

This paper reports a study that addresses the issue of adaptive expertise in the specific domain of multi-digit subtraction.

When solving multi-digit subtractions there are two specific strategies that can be distinguished based on the kind of arithmetic operation underlying their solution process: direct subtraction and subtraction by addition. Direct subtraction involves a subtraction operation to solve the subtraction, i.e. subtracting the subtrahend from the minuend (e.g., 712 – 346 = ?; 712 – 300 = 412, 412 – 40 = 372, 372 – 6 = 366), whereas in subtraction by addition the operation one performs to solve the subtraction is additive in nature, i.e. determining how much should be added to the subtrahend to obtain the minuend (e.g., 712 – 346 = ?; 346 + 54 = 400, 400 + 312 = 712, 54 + 312 = 366).

2 What Is Replicated?

Research on mental multi-digit subtraction by Torbeyns et al. (2018) shows that upper elementary school children frequently, efficiently, and adaptively used subtraction by addition. This is a remarkable finding, since textbook analyses and teacher interviews revealed that the mathematics instruction that these children received when in lower grades solely focused on direct subtraction.

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The paper by Van Der Auwera et al. (2021) reports a replication of the study by Torbeyns et al. (2018).

3 How Was the Replication Conducted?

Van Der Auwera et al. (2021) conducted an internal conceptual replication study. It is an internal replication, since three authors (Torbeyns, De Smedt and Verschaffel) were involved in both studies. It is a conceptual replication, since the research methodologies were similar but not the same. In addition to the obvious and necessary differences such as having a different sample of participating children, the main methodological changes were three:

1. *The use of optimized item series of subtraction problems*. Torbeyns et al. (2018) used items with a small difference (SD) between minuend and subtrahend (e.g., 614 – 596 = 18) and items with a large difference (LD) between minuend and subtrahend (e.g., 734 – 47 = 687). In the current study the difference of SD items and the subtrahend of LD items will always be between 12 and 28 in order to make the SD and LD items even more inviting to be solved using subtraction by addition and direct subtraction, respectively.
2. *The use of medium difference items as an additional type of experimental items*. The study by Torbeyns et al. (2018) included, next to SD and LD items, so called buffer items (e.g., 634 – 278 = 356). These buffer, or medium difference (MD), items consist of a three-digit minuend and subtrahend, a three-digit difference and the subtrahend and the minuend both have values close to half of the minuend. MD items were used to create variety in the item series, but were not analyzed. In Van Der Auwera et al. (2021) MD items were treated as a third type of experimental items and as such were included into the analyses.
3. *The introduction of a fourth condition, i.e., the choice-decide condition*. The study by Torbeyns et al. (2018) consisted of three conditions in which participants had to solve subtractions. One condition in which children were instructed to solve each subtraction with their preferential strategy, which is referred to as the *choice-compute* condition, and two conditions in which children were instructed to solve each subtraction with either direct subtraction or subtraction by addition, i.e., the *no-choice* conditions. In the current study a fourth and novel condition was included, called the *choice-decide* condition, in which children only had to indicate how they would solve each item, without being asked to actually do so.

4 Implications and Significance

Results show that the children participating in the current study made frequent, efficient and adaptive use of the untaught subtraction by addition strategy. Especially on the SD items, the use of this strategy was as accurate and quick as the systematically taught and intensively practiced direct subtraction strategy. This confirms findings from Torbeyns et al. (2018).

This new study yields additional support for the educational recommendations that have already been made by Torbeyns et al. (2018), particularly the recommendation to give the subtraction by addition strategy a much more prominent role in the teaching and practice of multi-digit subtraction.

The authors of this study share their instruments and empirical data (the experimental item set, the teacher questionnaire, the script and the collected and analyzed dataset can be accessed at 10.6084/m9.figshare.14420297) for possible further use. This openness contributes to establishing a culture of transparency and sharing, which is necessary for replication studies to flourish.

Reference

Torbeyns, J., Peters, G., De Smedt, B., Ghesquière, P. & Verschaffel, L. (2018). Subtraction by addition strategy use in children of varying mathematical achievement level: A choice/no-choice study. *Journal of Numerical Cognition*, *4*(1), 215–234. https://doi.org/10.5964/jnc.v4i1.77.