Impact Sheet

Problem Addressed

The paper addresses the well-known so-called Students-Professors (S/P) problem, which was first formulated over forty years ago by Kaput and Clement (1979). The paper reviews the extant research on this problem, which did not lead to a complete and exhausting understanding of its nature. It next discusses the problem in terms of mathematics modelling. More precisely, the authors operate with two versions of the problem: a mathematization version similar to the original formulation of the S/P-problem – i.e. "to write an algebraic equation, using S for the number of students and P for the number of professors, that is equivalent to the following statement: 'At a certain university, there are six times as many students as there are professors.'" (p. 208) – and a de-mathematization version of the problem, where students are asked to make sense of the expression 6P=S. The paper presents an empirical study of Danish upper secondary students' answers to the two versions of the S/P-problem; 296 student answers to the mathematization version and 658 student answers to the de-mathematization version. Writing 6S=P instead of 6P=S has been named the "reversal error".

Based on the data from these two (different) groups of Danish upper secondary school mathematics students, the authors ask the following questions: (1) What types of erroneous mathematical relationships and other expressions do students provide to the mathematization version of the S/P-problem (and what might possible reasons for these be)?; and (2) What types of erroneous interpretations do students offer to the de-mathematization version of the S/P-problem (and what might possible reasons for these be)?

What is Replicated?

The study reproduces several of the previous findings related to the S/P-problem that have been reported in the research literature approximately from the years 1980–1995. In addition, it expands these with new nuances and new findings – both from the original perspectives of the previous studies and from the new perspective of mathematical modeling – some of which offer new explanations of previous findings. To each of the two versions of the S/P-problem, the authors provide extensive classifications of error types and a large number of examples to illustrate these. For the mathematization version of the S/P-problem, the authors identify four major error types, the reversal error being one of them. For the de-mathematization version, the authors distinguish eleven major error types, two of which are instantiations of the reversal error.

The study may be described as a "conceptual replication" (e.g., Aguilar, 2020) that involves a methodological variation, e.g., in conditions and procedures, to test scope and generalizability of previous findings. Following the suggestion of Star (2018), the authors distinguish between "truth" and "understanding" in relation to their replication study, thus arguing that: "Although we do to some degree confirm the 'truth', although not the completeness, of several previous findings related to the S/P-problem, the main aspiration and contribution of the study described in this paper is to further 'understanding' of the phenomena and factors involved."

Implications and Significance

The mathematical modeling perspective adopted, along with a mixed-methods design, give rise to new potential explanations of the so-called reversal error as well as explanations of the new error types. The study shows that interpreting the linguistic formulation of the S/P-problem statement is not only related to language elements, but is intrinsically related to individuals' mathematico-cognitive models of mathematical statements cast in natural language.

In this sense, the study offers new and further insights to supplement the already vast research literature on the S/P-problem.

References

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