## Figures

These are our original figures. Note that they are different from the figures in the manuscript PDF file, because these were converted by the publisher to CMYK (with printer-specific parameter settings) so they look rather different from the RGB-originals on your display. It makes a difference for the images of stimuli, these are best judged from the figures in this file.


Figure 1: Typical stimuli harvested from the literature on "color harmony." These all have specific advantages and disadvantages for psychophysical or experimental phenomenological research. There is amply enough material for a voluminous review paper here. The stimuli used in this paper are categorically different.


Figure 2: Paul Sérusier (1864-1927), Le Talisman (1888, under guidance of Paul Gauguin). This was the key image to the Nabis (1888 until 1900), hence the title (original title "The Bois d'Amour at Pont Aven."). Art historically, it represents a very early example of almost completely chromatic painting (KWYBR). It looks very similar when put upside down, although the compositional balance varies with the orientation. Sérusier painted it only on the cover of a cigar box.


Figure 3: Colors used by the participants, mapped on the standard set. The piechart gives a visual impression of the frequency of occurrence. (Note that this graph pools likes and dislikes.)


Figure 4: Pie-charts of likes/dislikes for the whole group of participants.


Figure 5: Gender and age preferences


Figure 6: A graph of the similarity relations in preferences of subgoups. The vertex size indicates (sub-)group size, the thickness of the edges reflects similarity. Notice that the similarity between genders and that between age-groups is relatively low. This perception is validated by formal statistics.

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Figure 7: All forty-two constrained subsets, collected on the basis of cardinality.


Figure 8: An example involving yellow-blue. At left pixels are defined as $\operatorname{RGBColor}[\alpha, \alpha, \beta]$, where $\alpha, \beta$ are drawn from a uniform distribution of the unit interval. At center we draw two such uniform variates $\{\alpha, \beta\}$, normalize such that $\left\{\alpha^{\prime}, \beta^{\prime}\right\}=\{\alpha, \beta\} /(\alpha+\beta)$ and define pixels as RGBColor $\left[\alpha^{\prime}, \alpha^{\prime}, \beta^{\prime}\right]$. Finally, at right, we draw four such uniform variates $\{\alpha, \beta, \gamma, \delta\}$, normalize such that $\left\{\alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}, \delta^{\prime}\right\}=\{\alpha, \beta, \gamma, \delta\} /(\alpha+\beta+\gamma+\delta)$ and define pixels as RGBColor $\left[\beta^{\prime}+\gamma^{\prime}, \beta^{\prime}+\gamma^{\prime}, \beta^{\prime}+\delta^{\prime}\right]$, that is an even blend of black (weight $\alpha^{\prime}$ ), white (weight $\beta^{\prime}$ ), yellow (weight $\gamma^{\prime}$ ) and blue (weight $\delta^{\prime}$ ). The latter blend is the one used in the texture patterns. The first method (left) is perhaps more typical for the mainstream literature. (The quantitative differences are best judged from histograms of RGB-coordinates (Supplement 2), but the visual effect is immediately apparent from these samples.)


Figure 9: Examples of texture patterns. From left to right: an achromatic pattern, a monochromatic pattern (red), a dichromatic pattern (red and green) and a trichromatic pattern (red, green and blue). As explained (fig. 8) all patterns include black and white. Such patterns are rather unlike typical patterns used in the literature (fig. 1), which tend to be closer to a bi-partite or tri-partite field of uniform areas.


Figure 10: Examples of mosaic patterns. From left to right: an achromatic pattern, a monochromatic pattern (red), a dichromatic pattern (red and green) and a trichromatic pattern (red, green and blue). As explained (fig. 8) all patterns include black and white. Like the textures, such patterns are also rather unlike typical patterns used in the literature (fig. 1). They also appear as very different from the texture patterns (fig. 9), although the nominal patterns (just a tuple of color names!) of figs. 9 and 10 are identical.


Figure 11: Some foreground-background examples. Notice that the foreground shape visually "belongs to the picture" because of the lost-and-found edge-quality. The reason is that we attempt to force observers to look at the pattern as a whole. (One never knows, but at least on can try.) From left to right: Magenta foreground on yellow-cyan background, Yellow foreground on magenta-cyan background, and Cyan foreground on yellow-magenta background. Note all images involve the triple Cyan-Magenta-Yellow. Yet how different they look! We use such examples to suggest that the nominal designation Cyan-Magenta-Yellow is perhaps wanting in explanatory power. Picture sets like this almost enable "instant psychophysics" (Richards, 1987).


Figure 12: At top-left the six-step color circle, at bottom-left the neutral hues. At top centre and right the pure warm and cool hues, at bottom centre and right the augmented (with the neutral) warm and cool hues.


Figure 13: Eugène Brouillard (1870-1950), Arbres en bordure d'un chemin. Perhaps an extreme example (though common enough!) of the "teal-and-orange" palette. It is certainly not "natural" in the impressionist sense. Even such a minimalist palette is nevertheless "complete." This is where most of the chromatic action is. Other hues could provide "accents," it might work well enough in sufficiently small doses.


Figure 14: The various combinations used in the experiment. Here we do not show 18 "skew" combinations that do not naturally fit a conventional chord. In the boxes at top one has analogous schemes, from left to right: cool (top) or warm(bottom), augmented warm (top) or cool (bottom), neutral, and mixed, or perhaps "augmented neutral." Note mono,- di- and tri-chromatic instances. In the boxes at bottom, at left complementary pairs, either cool-warm (left), or neural (right). At right the two trichromatic chords. There are 23 classical chords and 18 skew combinations, thus about fifty-fifty. When the achromatic pair is added, there are 42 combinations in total.


Figure 15: A screen grab of the experiment at a moment that all responses were completed, so the yellow square is presented. Clicking the yellow square will initiate the next trial.


Figure 16: The overall use of the Likert scale.


Figure 17: Left: Overall counts over all participants and color combinations show that there might be some variation in the seven categories. Right: A dendrogram reveals the nature of mutual clustering among the categories.


Figure 18: Randomly splitting the participants in two halves and comparing the groups on particular color combinations and categories reveal a fairly strong dependence. (Here 0.0 is the neutral level, $\pm 1$ a single step up or down in the Likert scale $(-2,+2)$.) This is just one random case, but we find that it is typical for any random partition. It indicates that the data is systematically structured.


Figure 19: This is a histogram of pairwise correlations in two groups of participants, a "concordant" group and the remainder. The "concordant" group members are highly correlated among each other. The median is 0.63 , interquartile range $0.54-0.72$. Members of the remainder group are at best weakly correlated with the others. The median is 0.31 , interquartile range $0.10-0.46$. Thus the "concordant" and "remainder" groups are really different and have only minimal overlap. It may not be an (in some sense) optimal bipartition, but it is certainly effective.

calm, soothing, subdued
calm, soothing, subdued


Figure 20: Profile clusters for the concordant group. Only cluster with two or more members are shown. (Note that we omit the white-black in chromatic combinations. Remember that all combinations contain Kw.)


Figure 21: Word cloud profile responses for the concordant group. Here we show the profiles of the combinations that evoked the strongest responses.


Figure 22: Pairwise correlation matrix of response profiles for all color combinations. This is for the concordant group. More orange indicates more positive (the diagonal has the maximum value), whereas more teal indicates more negative values. Insignificant correlations are near white. There is much "noise," two singular values explain $90 \%$ of the variance.


Figure 23: Result of clustering using the K-Means algorithm with Euclidean distance and standard deviation criterion function, asking for two clusters. Notice that the two clusters (top row and bottom row) split perfectly according to the traditional cool-warm criterion. The clustering was on the responses of the concordant group for all color combination that were either cool or warm.


Figure 24: Monochrome responses for the dominant category as a function of the hue order in the color circle for the concordant group. Note that there appears to be a smooth transition in the magnitude of the response. Compare fig. 25. The "gloomy" response to cyan is still seen in the word cloud for blue, but is (almost) absent in the word cloud for green.


Figure 25: Monochrome responses as a function of hue for the concordant group. Compare fig. 24, that gives the strengths of the responses and fig. 26 that shows pairwise clustering tendencies.


Figure 26: Dendrogram for the monochromatic profiles for the concordant group. Cool and warm are nicely separated at the first node. The other nodes are on the warm side of the tree. Green is seen to split off from yellow, purple from red. (Compare also fig. 25.) The resulting pairs (lowermost levels) are very common choices for analogous schemes (see also fig. 27).


Figure 27: Some dichromatic patterns suggested by the dendrogram of monochromatic schemes (fig. 26) for the concordant group.


Figure 28: Left analogous cool, right analogous warm combinations for the concordant group. Widening the range makes very little difference. (The numbers are the average responses on the $(-2,+2)$ Likert scale.)


Figure 29: The complementary (left) and triadic (right) schemes for the concordant group. Why the jump from "gloomy," "irritating," to "lively," "cheerful?" Of course, our data cannot provide an answer to that. (The numbers are the average responses on the $(-2,+2)$ Likert scale.)


Figure 30: Word clouds for the monochromatic mosaics. (Compare fig. 25. Note that only the relative size is important in the word clouds, don't take letter size as absolute.)


Figure 31: Dendrogram for the foreground/background stimuli. The clustering is evidently by color tuplets, irrespective the foreground/background relation.


Figure 32: Bar chart for the comparison of red-cyan and cyan-red. The dashed levels indicate half a Likert-scale step. Although the pairs are the same, the foreground/background relation makes a difference. ("C-R" denotes " R foreground, c background" and so forth.)


Figure 33: The distinct patterns for the triple MYC.


Figure 34: At left, Delacroix 1834 (from a notebook). At right, Paul Klee, Kanon der farbigen Totalität, 1931 (drawing from "Bildnerische Gestaltungslehre," Zentrum Paul Klee, Bern). The orange and violet patches are noticeable, but the green patch is quite vague - possibly the paint bleached over the decades. Notice the difference between "G" (yellow, from Gelb) and "gr" (green, from Grün).


Figure 35: Van Doesburg and Rietveld interior, 1919. Black, gray, white, blue, yellow and red.


Figure 36: Histograms of correlations of responses of pairs of participants over all combinations. This involves all 60 participants.


Figure 37: The dendrogram of all data summarized by equating Red with Magenta, Green with Yellow and Cyan with Blue.


Figure 38: The simplified correlation matrix (compare fig. 22).


Figure 39: Median pair-correlations for all coarse-graining schemes. The proposed scheme leads to the highest overall concordance, it is a clear winner. This involves all 60 participants. Lower correlations interchange (near) complementaries like red and cyan.


Figure 40: Red (top row), blue (centre row) and yellow (bottom row). Coarsegraining is something to get used to. However, the row-wise family resemblance is clear enough. All these patterns are rated "strong," but with different annotation: blues are "gloomy," reds are "intrusive" and yellows are "intrusive" to "irritating."


Figure 41: The remaining combination families restricted to BRY. There are 32 instances here, more than three-quarters of all combinations. From top to bottom these are predominantly "intrusive, strong, vivid," "lively, strong, vivid," "piercing, strong, vivid" and " lively, strong, vivid."


Figure 42: Left a color triangle by Philip Otto Runge (Runge, 1810), right one from Charles Blanc's Grammar of Art (Blanc, 1891). These schemes are by no means "hollow," like the triangle of Delacroix (fig. 34).

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Figure 43: Bisections of the daylight spectrum according to Schopenhauer (Schopenhauer, 1816). (The top line has: "the following scheme derives from my representation:".) Such bisections are necessarily complementary. Thus the blue (Blau) must be teal. The red must be not too different from magenta. The violet (here spectral!) must be close to blue. This maps directly on fig. 42.


Figure 44: The Goethe edge colors (Kantenspektren). The cool edge colors are shades of blue, varieties of teal and tints of cyan, easily taken for "blue." The trisection shown here splits the full cool edge color series into these three subgroups. The warm edge colors are tints of yellow, varieties of orange and shades of red, easily taken for "yellow and red." The trisection shown here splits the full warm edge color series into these three subgroups. Looking at "all colors" in this way suggests that if there are any primaries they should be blue, yellow and red.


Figure 45: The green is a subtractive mixture of yellow and cyan ("blue"), the purple a partitive mixture of red and blue.


Figure 46: Arnold Böcklin (1827-1901), Ruggiero und Angelica Anagoria, 1873. A rare example of a KWRYC combination, straddling the neutral axis (RY against C). Colors have been assigned to actors, with minor cross-overs (e.g, the claws of the beast, the plumage on Ruggiero 's helmet) that are important in the overall composition. This color combination would hardly look good in a texture.


Figure 47: Félix Vallotton (1865-1925 ), Rocher à Ploumanach, 1917. A perfect example of the Blue-Yellow combination. A very strong pattern. Lively, vivid, irritating, cheerful, intrusive, piercing? Some of these, rather than their opposites. We'd probably pick "vivid," or "lively." How much depends upon the topic? Does it work out the same upside down? Stimuli like those in fig. 1 hardly address such issues.


Figure 48: Sérusier, Le forêt sacrcé (1891). An extreme yellow-red combination. What about such a scheme? Perhaps "piercing and irritating?" Ratings are bound to differ. The nominal subject may influence the choice.


Figure 49: Franz Marc (1880-1916), Kämpfende Formen (1914.). A blue-red combination with a yellow accent. The turmoil of the aggressive shapes changes the affective value of the colors here. How would one rate it? "Intrusive," or "irritating" perhaps? Certainly not "dull," but perhaps a touch of "calm?" Up to the viewer. Knowing the title will likely make a difference. Knowing about Marc will make another.


Figure 50: Maurice Denis (1870-1943), Le Paradis, 1912. A rare example of a purple (tint) - teal (shade) combination. Yellow and pale cyan are extensive, red only punctate accents. These are important in the composition, but not decisive for the color scheme. Together it makes for a perhaps tetrachromatic scheme. How might it be rated? Calm, subdued, or soothing appear to apply, whereas gloomy, piercing, or irritating certainly do not. It seems to us that such ratings are at least not completely arbitrary. They are not very determinate either.


Figure 51: Example stimulus. This image is an equal mixture of black, white, cyan, magenta and yellow. It is completely random, thus yields rich material for the imagination. No participant was ever bored.

